20th Workshop on Hereditary Graph Properties

Piechowice, Poland

September 22–24, 2017

HEREDITARNIA

Foreword

20 years ago mathematicians from Košice, Pretoria and Zielona Góra started an informal, regular series of meetings called "Hereditarnia Workshops" that focussed on hereditary properties of graphs.

Although that focus became less pronounced over the years, the meetings brought together mathematicians with similar interests, and became a catalyst for collaboration.

We thank all participants for their interest in the workshop and for their contributions to the scientific program, and wish you a pleasant and productive sojourn, maybe even the beginning of a new and interesting collaboration!

We also wish the workshop on the occasion of its 20th birthday a cordial "sto lat".

Wilfried Imrich and Elżbieta Sidorowicz

Leoben and Zielona Góra, September 2017

Program

Friday 22.09.2017

15:00 – 15:10	Opening and welcome address
15:10 – 15:30	Mieczysław Borowiecki and Izak Broere
15:30 – 16:20	Rafał Kalinowski

16:20 – 17:00 Coffee Break

Chair:	Anna Fiedorowicz
17:00 – 17:30	Gabriel Semanišin
17:30 – 17:50	Marta Borowiecka-Olszewska

18:00 – 18:30 Annual meeting of the Hereditarnia Club

19:00 – 21:00 Dinner

Saturday 23.09.2017

7:00 – 9:00 Breakfast

Chair:	Sandi Klavžar
9:30 – 10:20	Mieczysław Borowiecki
10:30 – 10:50	Boštjan Brešar

10:50 – 11:30 Coffee Break

Chair:	Mariusz Woźniak
11:30 – 11:50	Agata Drzystek
12:00 – 12:20	Harald Gropp
12:30 – 12:50	Iztok Peterin

13:00 – 14:00 Lunch

Free time

17:00 – 18:30 Discussion about hereditary problems

19:00 – 21:00 Conference Dinner

Sunday 24.09.2017

7:00 – 10:00 Breakfast

Depart

Abstracts

Abstracts are listed alphabetically with respect to the PRESENTING AUTHOR.

ON PERFECT CONSECUTIVELY COLOURABLE GRAPHS

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A *consecutive colouring* of a graph is a proper edge colouring with natural numbers in which the colours of edges incident with each vertex form an interval of integers. The idea of this colouring was introduced in 1987 by Asratian and Kamalian [1] under the name of *interval colouring*. Sevastjanov showed that the corresponding decision problem is NP-complete even restricted to the class of bipartite graphs. The class $\mathfrak{N}$ of consecutively colourable graphs is not induced hereditary.

We focus our attention on the class of consecutively colourable graphs whose all induced subgraphs are consecutively colourable, too. We denote this class by $\mathfrak{N}_{\leq}$ and call their elements *perfect consecutively colourable* graphs to emphasise the conceptual similarity to perfect graphs. Obviously, the class of perfect consecutively colourable graphs is induced hereditary, so it can be characterized by the family $\mathcal{C}(\mathfrak{N}_{\leq})$ of induced forbidden graphs. In this work we give some properties of graphs in $\mathcal{C}(\mathfrak{N}_{\leq})$. Moreover, we show necessary and sufficient conditions that must be satisfied by a graph to be in $\mathcal{C}(\mathfrak{N}_{\leq})$, for some families of graphs.

Keywords: consecutive colourings, deficiency, Sevastjanov graphs, forbidden graphs.

AMS Subject Classification: 05C75, 05C15, 05C35.

References

[1] A.S. Asratian and R.R. Kamalian, Interval colorings of the edges of multigraph (in Russian), Appl. Math. **5** (1987) 25–34.

THE 1-2-3 CONJECTURE FOR $\mathcal{P}$-COLOURINGS

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We consider finite undirected graphs without loops or multiple edges. Let $\mathcal{I}$ denote the class of all graphs. A *property of graphs* is any nonempty class of graphs from $\mathcal{I}$ which is closed under isomorphisms.

Let $\preceq$ be a partial order on the set $\mathcal{I}$. A property $\mathcal{P}$ is said to be $\preceq$-*hereditary* if whenever $G \in \mathcal{P}$ and $H \preceq G$, then also $H \in \mathcal{P}$.

A property $\mathcal{P}$ is *induced hereditary* if it is $\leq$-hereditary with respect to the relation "$\leq$" *to be an induced subgraph* and $\mathcal{P}$ is *hereditary* if it is $\subseteq$-hereditary with respect to the relation "$\subseteq$" *to be a subgraph.*

A property $\mathcal{P}$ is *additive* if the disjoint union $H \cup G \in \mathcal{P}$ whenever $G \in \mathcal{P}$ and $H \in \mathcal{P}$.

Examples of additive induced hereditary properties:

$$\begin{array}{lcl} \mathcal{O} & = & \{G \in \mathcal{I} : E(G) = \emptyset\}, \\ \mathcal{D}_1 & = & \text{the class of all acyclic graphs,} \\ \mathcal{D}_k & = & \text{the class of all } k\text{-degenerate graphs,} \\ \mathcal{S}_k & = & \text{the class of all graphs } G \text{ with } \Delta(G) \leq k. \end{array}$$

A *k-edge labelling* of a graph G is a function $l : E(G) \to \{1, \ldots, k\}$. Every such labelling induces a vertex colouring c defined, for each vertex v of G, by the summation of the labels of the edges incident to v, i.e., $c(v) = \sum_{e\ inc\ v} l(e)$ for each v of G. We call the function c the *vertex colouring induced by the k-edge labelling* (for short, the *induced colouring*). The set of vertices $V_i = \{v \in V(G) : c(v) = i\}$ is called a *colour class.* Let $\mathcal{P}$ be an additive induced hereditary property. If, for the induced colouring c of an edge labelling l, every colour class V_i induces a subgraph $G[V_i]$ which is in $\mathcal{P}$, then we call c an *induced $\mathcal{P}$-colouring of G.* The minimum k for which G has an induced $\mathcal{P}$-colouring is denoted by $\chi^{\mathcal{P}}_{\Sigma}(G)$.

The study of these parameters started with $\mathcal{P} = \mathcal{O}$. Indeed, in [2] the following conjecture is formulated:

Conjecture 1 *Let G be a connected graph with $G \neq K_2$. Then $\chi_\Sigma^{\mathcal{O}}(G) \leq 3$.*

In the literature, the main focus has been on the case $\mathcal{P} = \mathcal{O}$; see for instance the survey of 2012 ([3]) in this regard. However, a recent "relaxed case" has been studied in [1] where it is shown amongst others that $\chi_\Sigma^{\mathcal{D}_1}(G) \leq 3$ for every graph G while $\chi_\Sigma^{\mathcal{D}_1}(G) \leq 2$ if G is a graph with maximum average degree at most 3 or G is a series-parallel graph.

The above results and the results we offer contribute to our reason for formulating the following extension of the original 1-2-3 Conjecture.

Conjecture 2 *For every connected graph $G \neq K_2$,*

$$\chi_\Sigma^{\mathcal{P}}(G) \leq \begin{cases} 3, & \mathcal{P} = \mathcal{O}, \\ 2, & \textit{otherwise}. \end{cases}$$

Keywords: 1-2-3 Conjecture, $\mathcal{P}$-colouring.

AMS Subject Classification: 05C15.

References

[1] Y. Gao, G. Wang and J. Wu, A relaxed case on 1-2-3 Conjecture, Graphs Combin. **32** (2016) 1415–1421.

[2] M. Karoński, T. Łuczak and A. Thomason, Edge weights and vertex colours, J. Combin. Theory Ser. B **91** (2004) 151–157.

[3] B. Seamone, The 1-2-3 Conjecture and related problems: a survey. arXiv: 1211.5122v1 math[CO] 21 November 2012

CONVEX AND ISOMETRIC DOMINATION OF CLASSES OF (CHORDAL) GRAPHS

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A set of vertices D in a graph G is a *convex dominating set* of G if D is a dominating set such that all vertices on all shortest paths between any two vertices in D lie in D. The study of convex domination was initiated in 2004 by Lemańska [1] and Raczek [3]. It is known (see [3]) and easy to see that given a graph G and a positive integer k it is NP-hard to determine whether G has a convex dominating set of size at most k, even when G is restricted to bipartite or split graphs (and hence also if G is restricted to chordal graphs). In this talk, we consider algorithmic aspects of convex domination, and of its variation that we call isometric domination.

A pair of vertices x, y in a graph G such that the vertices of any path between x and y form a dominating set is a *dominating pair*, and graphs that contain a dominating pair are *weak dominating pair graphs*. (For instance, the well-known interval graphs belong to this class.) It is shown that even if one restricts to chordal weak dominating pair graphs the convex domination problem remains NP-complete. On the other hand, for a version of its hereditary subclass (called *chordal hereditary dominating pair graphs* in [2]) a polynomial time algorithm determining the minimum size of a convex dominating set for graphs in this class is presented.

Keywords: convex domination, dominating pair graph, convex hull.

AMS Subject Classification: 05C69, 05C85.

References

[1] M. Lemańska, Weakly convex and convex domination numbers, Opuscula Math. **24** (2004) 181–188.

[2] N. Pržulj, D. Corneil and E. Köhler, Hereditary dominating pair graphs, Discrete Appl. Math. **134** (2004) 239–261.

[3] J. Raczek, NP-Completeness of weakly convex and convex dominating set decision problems, Opuscula Math. **24** (2004) 189–196.

$\mathcal{D}_1$-SUM-LIST COLOURINGS

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A vertex colouring of a graph G is called *acyclic* if the subgraph induced by each colour class is a forest. Let f be a function on the vertex set of a graph G with natural values. The graph G is $(f, \mathcal{D}_1)$-*choosable* if for every collection of lists with sizes specified by values of f there is an acyclic colouring of G using colours from the lists. The $\mathcal{D}_1$-*sum-choice-number* of a graph G is the minimum of the sum of sizes in f over all f such that G is $(f, \mathcal{D}_1)$-choosable.

Using the notions defined above, we present some results on $\mathcal{D}_1$-sum-choice-numbers of 2-trees and cylinders.

References

[1] E. Drgas-Burchardt and A. Drzystek, General and acyclic sum-list-colouring of graphs, Appl. Anal. Discrete Math. **10** (2016) 479–500.

[2] E. Drgas-Burchardt and A. Drzystek, Acyclic sum-list-colouring of grids and other classes of graphs, Opuscula Math. **37** (2017) 535–556.

DISTINGUISHING COLOURINGS OF GRAPHS

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We consider general, i.e. not necessarily proper, colourings of graphs. A colouring of a graph G is *distinguishing* if the only automorphism of G preserving colours is the identity. The minimum number of colours in a vertex, edge or total distinguishing colouring of G is called the *distinguishing number* $D(G)$, the *distinguishing index* $D'(G)$ or the *total distinguishing number* $D''(G)$ of a graph G, respectively. These invariants were defined consecutively in [1], [2] and [3].

Tons of papers by numerous authors have been published, especially in the last dozen of years, and this is still a growing area of discrete mathematics. Distinguishing colourings have applications in computer science, in particular in distributed computing.

In the talk, some recent results joint with W. Imrich, M. Pilśniak, T. Tucker, M. Woźniak et al. will be presented.

Keywords: symmetry breaking in graphs, graphs automorphisms, colourings of graphs.

AMS Subject Classification: 05C15, 05C25, 05C35.

References

[1] M.O. Albertson and K.L. Collins, Symmetry breaking in graphs, Electron. J. Combin. **3** (1996) R18.

[2] R. Kalinowski and M. Pilśniak, Distinguishing graphs by edge-colourings, European J. Combin. **45** (2015) 124–131.

[3] R. Kalinowski, M. Pilśniak and M. Woźniak, Distinguishing graphs by total colourings, Ars Math. Contemp. **11** (2016) 79–89.

CARTESIAN PRODUCTS OF DIRECTED GRAPHS WITH LOOPS

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We show that every nontrivial finite or infinite connected directed graph with loops and at least one vertex without a loop is uniquely representable as a Cartesian or weak Cartesian product of prime graphs. For finite graphs the factorization can be computed in linear time and space. Moreover, by the use of data structure developed in [2], the algorithm for factorization of digraphs is a significant simplification of the first such linear algorithm from [1]. The algorithm gains its simplicity by the hereditary behavior of orientation of the edges by projections to the factors.

Keywords: directed graph with loops, infinite graphs, Cartesian and weak Cartesian products.

AMS Subject Classification: 05C25, 05C20.

References

[1] Ch. Crespelle and E. Thierry, Computing the directed Cartesian-product decomposition of a directed graph from its undirected decomposition in linear time, Discrete Math. **3**38 (2015) 2393–2407.

[2] W. Imrich and I. Peterin, Recognizing Cartesian products in linear time, Discrete Math. **3**07 (2007) 472–483.

CONCEPTS AND PROBLEMS OF HEREDITARNIA

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The language of hereditary properties of graphs provides a powerful tool for a systematic study of one of the central problem of the Graph Theory — generalised colouring.

The first attempt to present a complex overview of this approach was made in [1]. However, the complex terminology and systematisation of known results was presented a little bit later in [2]. This survey stands at the beginning of a series of papers on various problems related to hereditary properties of graphs. It is worth to mention that two important hereditary property problems were already included in the famous book [3] that was published two years before the survey.

We briefly summarise the main concepts and problems of Hereditarnia. We also present some open problems and try to indicate a possible directions for further research in Hereditarnia.

Keywords: path vertex cover number, decycling number, P_4-tidy graph.

AMS Subject Classification: 05C35.

References

[1] M. Borowiecki and P. Mihok, Hereditary properties of graphs, in: Advances in Graph Theory, ed. V.R. Kulli, Vishwa International Publications, 1991, 42–69.

[2] M. Borowiecki, I. Broere, M. Frick, P. Mihók and G. Semanišin, A survey of hereditary properties of graphs, Discuss. Math. Graph Theory **17** (1997) 5–50.

[3] T.R. Jensen and B. Toft, Graph Coloring Problems, John Wiley & Sons, 1995.

[1]The research was partially supported under the contract VEGA 1/0142/15

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